A New Look At $\mathbb{R}$

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A Suggestion

Let G: $\mathbb{R} \to \mathcal{P}(\mathrm{S1})$ be defined through

$$\forall x \in \mathbb{R} \quad \mathrm{G}(x) := \underbrace{\{y \in \mathrm{S1}: \exists t \in \mathbb{Z} \quad y=\exp(2\pi i(t \cdot x))\}}_{\subseteq \mathbb{C}}$$

Theo.:

Ass.: $\forall x \in \mathbb{R} \quad \begin{pmatrix} x \in \mathbb{Q} \quad \Leftrightarrow \\ ((\mathrm{G}(x), \cdot) \text{ is a finite subgroup of } (\mathrm{S1}, \cdot)) \end{pmatrix}$

Theo.:

Ass.: $\forall x \in \mathbb{R} \quad \begin{pmatrix} x \in \mathbb{Z} \quad \Leftrightarrow \\ \mathrm{G}(x) = \{1\} \end{pmatrix}$

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Ass.: $\forall x \in \mathbb{R} \quad \begin{pmatrix} x \in \mathbb{Q} \quad \Leftrightarrow \\ \begin{pmatrix} \mathbb{Z} & \to & \mathrm{S1} \\ t & \mapsto & \exp(2\pi i(t \cdot x)) \end{pmatrix} \text{ is not injective} \quad \Leftrightarrow \\ \#(\mathrm{G}(x)) < \infty \end{pmatrix}$

We define a map D: $\mathbb{R} \to \mathbb{N}_+ \cup \{\infty\}$ through

$$\forall x \in \mathbb{R} \quad \mathrm{D}(x) := \#(\mathrm{G}(x))$$

D is a determinant for rational numbers

$\mathbb{R}$ is a bunch of Spheres

We define a map $\varphi\colon\ \mathbb{R} \to \mathbb{Z} \times [0;1[$ through

$$\forall t \in \mathbb{R}\ \ \varphi(t) := \begin{cases} \left([t]_g\ ; t - [t]_g\right) & \text{t>=0} \\ \left(-[-t]_g\ ; t + [-t]_g\right) & \text{t<=0} \end{cases}$$

$\left([\ldots]_g \text{ is the Gaussian floor function}\right)$

Than the following is true:

$$\forall t \in \mathbb{R}\quad t = \varphi_1(t) + \varphi_2(t)$$

We define a map $\Phi\colon\ \mathbb{R} \to \mathbb{Z} \times S^1$ through

$$\forall t \in \mathbb{R}\quad \Phi(t) := \left(\varphi_1(t)\ ; \exp\left(2\pi\, i \cdot \varphi_2(t)\right)\right)$$

Then the following is true:

$\Phi\colon\ \mathbb{R} \to \mathbb{Z} \times S^1$ is a bijection

Because $\mathbb{Z} \times S^1$ is a commutative group, you get a new map $\odot\colon\ \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ with the properties

$(\mathbb{R};\odot)$ is a commutative group
$(\mathbb{Q};\odot)$ is a subgroup of $(\mathbb{R};\odot)$

Perhaps it was a good idea to look at a map $\tilde{\varphi}\colon\ \mathbb{R} \to \mathbb{Z} \times [0;1[$

$$\forall t \in \mathbb{R}\ \ \tilde{\varphi}(t) := \left([t]_g\ ; t - [t]_g\right)$$

Tunneling

We define a map $\lambda\colon \mathbb{R} \setminus \mathbb{Z} \to \mathbb{C}$ through

$$\forall t \in \mathbb{R} \setminus \mathbb{Z} \quad \lambda(t) := \exp\left(2\pi i \cdot \tilde{\varphi}_2(t)\right) + 2\tilde{\varphi}_1(t)$$

We define a set $\Lambda \subseteq \mathbb{C}$ through

$$\Lambda := \bigcup_{z \in \mathbb{Z}} S^1(2z)$$

$S^1(2z)$ is the unity sphere with center $2z$

Than the following is true:

$\lambda\colon \mathbb{R} \setminus \mathbb{Z} \to \Lambda$ is a bijection